II. *A Diſcourſe concerning a Method of Diſcovering the true Moment of the Sun's Ingreſs into the Tropical Signs. By* E. Halley.

IT may perhaps paſs for a Paradox, if not ſeem extravagant, if I ſhould aſſert that it is an eaſier matter to be aſſured of the moments of the Tropicks, or of the times of the *Sun's* entrance into *Cancer* and *Capricorn*, than it is to obſerve the true times of the Equinoctials or Ingreſs into *Aries* and *Libra*. I know the Opinion both of Ancient and Modern Aſtronomers to the contrary; *Ptolemy* ſays expreſly, Τὰς τῶν τροπῶν τηρήσεις δυσδιακρίτους εἶναι; And *Ricciolus* begins his Chapter of the Solſtitial Obſervations with theſe words, *Merito* Snellius,*in notis ad obſervationes* Haſſiacas, *pronunciavit, Herculei eſſe laboris vitare in* Solſtitiis *obſervandis errorem quadrantis diei*, and this becauſe of the exceeding ſlowneſs of the change of the Sun's Declination on the day of the Tropick, being not a quarter of a Minute in 24 Hours. This indeed would make it very difficult, nor would any Inſtruments ſuffice to do it, were the moment of the Tropick to be determined from one ſingle Obſervation. But by three ſubſequent Obſervations made near the Tropick,at proper intervals of time, I hereby deſign to ſhew a Method to find the moment of the Tropicks capable of all the exactneſs the moſt Accurate can deſire; and that without any conſideration of the Parallax of the Sun, of the Refractions of the Air, of the greateſt Obliquity of the Ecliptick, or Latitude of the Place: All which are required to aſcertain the times of the Equinoctials from Obſervation, and which being faultily aſſumed, have occaſioned an Error of near three Hours in the times of the Equinoctials deduced from the Tables of the Noble *Tycho Brahe* and *Kepler*,

Kepler, the Vernal being ſo much later, and the Autumnial ſo much earlier than by the *Calculus* of thoſe Famous Authors.

Now before we proceed, it will be neceſſary to premiſe the following *Lemmata*, ſerving to demonſtrate this Method, *viz.*

1. That the motion of the Sun in the Ecliptick, about the time of the Tropicks, is ſo nearly equable, that the difference from equality is not ſenſible, from five days before the Tropick, to five days after: and the difference ariſing from the little inequality that there is, never amounts to above $\frac{1}{4}$ of a ſingle Second in the Declination, and this by reaſon of the nearneſs of the *Apogæon* of the *Sun* to the Tropick of *Cancer*.

2. That for five Degrees before and after the Tropicks, the differences whereby the Sun falls ſhort of the Tropicks, are as the Verſed Sines of the Sun's diſtance in Longitude from the Tropicks; which Verſed Sines in Arches under five Degrees, are beyond the utmoſt nicety of ſenſe, as the Squares of thoſe Arches. From theſe two follow a Third:

3. That for five days before and after the Tropicks, the Declination of the Sun falls ſhort of the utmoſt Tropical Declination, by Spaces which are in duplicate proportion, or as the Squares of the Times by which the Sun is wanting of or paſt the moment of the Tropick.

Hence it is evident that if the Shadows of the Sun, either in the Meridian or any other Azimuth, be carefully obſerved about the time of the Tropicks, the Spaces whereby the Tropical ſhade falls ſhort of, or exceeds thoſe at other times, are always proportionable to the Squares of the Intervals of Time between thoſe Obſervations and the true time of the Tropick, and conſequently if the Line, on which the Limits of the ſhade is taken, be made the Axis, and the correſpondent times from the Tropick expounded by Lines, be erected on their

their reſpective Points in the Axis as ordinates, the extremities of thoſe Lines ſhall touch the Curve of a Parabola; as may be ſeen in the Figure: Where a, b, c, e, being ſuppoſed Points obſerved, the Lines aB, bC, cA, eF, are reſpectively proportional to the times of each Obſervation before or after the Tropical Moment in *Cancer*.

This premiſed, we ſhall be able to bring the Problem of finding the true time of the Tropick by three Obſervations, to this Geometrical one: having three Points in a Parabola A, B, C, or A, F, C given, together with the direction of the Axis, to find the Diſtance of thoſe Points from the Axis. Of this there are two caſes, the one when the time of the ſecond Obſervation B is preciſely in the middle time between A and C: In this caſe putting t for the whole time between A and C, we ſhall have Ac the interval of the remoteſt Obſervation A from the Tropick by the following Analogy.

As 2ac—bc to 2ac—½bc :: So is ½t or AE : to Ac the time of the remoteſt Obſervation A from the Tropick.

But the other caſe when the middle Obſervation is not exactly in the middle between the other two times, as at F, is ſomething more operoſe, and the whole time from A to C being put = t, and from A to F = s, ce = c, and bc = b, the Theorem will ſtand thus $\frac{ttc - bss}{2tc - 2bs} = Ac$ the time ſought.

To illuſtrate this Method of Calculation it may perhaps be requiſite to give an Example or two for the ſake of thoſe Aſtronomers that are leſs inſtructed in the Geometrical part of their Art.

Anno 1500 *Bernard Walther* in the Month of *June* at *Nuremburg* obſerved the Chord of the diſtance of the Sun from the *Zenith* by a large Parallactick Inſtrument of *Ptolemy*, as follows:

June 2. 45467. *June* 8. 44975.
June 9. 44934. and *June* 12. 44883.
June 16. 44990. *June* 16. 44990.

In both which caſes the middle time is exactly in the middle between the extreams, and therefore in the former three, a c = 533, b c = 477 and t, the time between being 14 days, by the firſt Rule, the time of the Tropick will be found by this Proportion, as 589 to $827\frac{1}{2}$: : So $\frac{1}{2}$ t or 7 days to 9 days $20^{h.}\ 2'$. whence the Tropick *Anno* 1500 is concluded to have fallen *June* $11^{d.}$ $20^{h.}\ 2'$. In the latter three, a c is = 107, and b c = 15, and the whole interval of Time is 8 days = to t; whence as 199 : to $206\frac{1}{2}$: : ſo is 4 days to $4^{d.}\ 3^{h.}\ 37'$. which taken from the 16*th*. day at Noon, leaves $11^{d.}\ 20^{h.}\ 23'$. for the time of the Tropick, agreeing with the former to the third part of an hour.

Again, *Anno* 1636, *Gaſſendus* at *Marſeilles* obſerved the Summer Solſtice by a *Gnomon* of 55 Foot high, in order to determine the Proportion of the *Gnomon* to the Solſtitial ſhade, and he hath left us theſe Obſervations, which may ſerve as an Example for the ſecond Rule.

June 19. *St. N.* ſhadow 31766 parts, whereof the *Gnomon*
June 20. 31753 (was 89428.
June 21. 31751
June 22. 31759

Theſe being divided into two ſetts of three Obſervations each, *viz.* the $19^{th.}$ $20^{th.}$ and $22^{th.}$ and the $19^{th.}$ $21^{th.}$ and $22^{th.}$ we ſhall have in the firſt three, c = 13 and b = 7, t = 3 days, s = 1, and in the ſecond c = 15 and b = 7, t = 3 and s = 2. Whence according to the Rule, the 19*th*. day at Noon the Sun wanted of the Tropick a time proportionate to one day, as t t c — s s b to 2 t c — 2 b s, that is, as 110 to 64 in the firſt ſett, or

107 to 62 in the ſecond ſet; that is, $1^{d.}\ 17^{h.}\ 15'$. in the firſt, or $1^{d.}\ 17^{h.}\ 25'$. in the ſecond ſett: So that we may conclude the Moment of the Tropick to have been *June* $10^{d.}\ 17^{h.}\ 20'$. in the Meridian of *Marſeilles.*

Now that theſe two Tropical times thus obtained, will be found to confirm each others exactneſs from their near agreement, appears by the interval of time between them; *viz.* $1^{d.}\ 2^{h.}\ 30'$. leſs than 136 *Julian* years: whereof $1^{d.}\ 1^{h.}\ 8'$. ariſes from the defect of the length of the Tropical Year from the *Julian*, and the reſt from the Progreſſion of the Sun's *Apogæon* in that time; ſo that no two Obſervations made by the ſame Obſerver in the ſame place, can better anſwer each other, and that without any the leaſt Artifice or Force in the management of them.

What were the Methods uſed by the Ancients to conclude the hour of the Tropicks, *Ptolemy* has no where delivered; but it were to have been wiſhed that they had been aware of this, that ſo we might have been more certain of the moments of the Tropicks we have received from them, which would have been of ſingular uſe to determine the Queſtion, Whether the Sun's *Apogæon* be fixt in the Starry Heaven; or if it move, what is the true motion thereof? It is certain, that if we take the Account of *Ptolemy*, the Tropick ſaid to be obſerved by *Euctemon* and *Meton*, *Junii* 27. *manè*, *Anno* 432 *ante Chriſtum*, can no ways be reconciled without ſuppoſing the Obſervation made the next day, or *June* 28*th.* in the Morning. And *Ptolemy*'s own Tropick obſerved in the Third Year of *Antoninus*, *Anno Chriſti* 140, was certainly on the 23*th.* and not the 24*th.* day of *June*; as will appear to thoſe that ſhall duly conſider and compare them with the length of the Year deduced from the diligent and concordant Obſervations of thoſe two great Aſtronomical *Genii*, *Hipparchus* and *Albatâni*; eſtabliſhed and confirmed by the concurrence of all the Modern Accuracy. For theſe Obſervations give the length of the Tropical Year ſuch as to anticipate the *Julian* Account only one day in 300 Years; but we are now ſecure that the ſaid Period of the Sun's Revolution does anticipate very nearly three days in 400 Years; ſo that the *Tables* of *Ptolemy* founded on that Suppoſition, do err about a whole day in the Sun's Place, for every 240 Years. Which principal Error in ſo Fundamental a Point, does vitiate the whole Superſtructure of the *Almageſt*, and ſerves to convict its Author of want of Diligence, or Fidelity, or both.

But to return to our Method, the great Advantage we have hereby, is, that any very high Building ſerves for an Inſtrument, or the top of any high Tower or Steeple, or even any high Wall whatſoever, that may be ſufficient to intercept the Sun, and caſt a true ſhade: Nor is the Poſition of the Plane on which you take the ſhade, or that of the Line therein, on which you meaſure the Receſs of the Sun from the Tropick, very material; but in what way ſoever you diſcover it, the ſaid Receſs will be always in the ſame Proportion, by reaſon of the ſmallneſs of the Angle, which is not ſix Minutes in the firſt five days: Nor need you enquire the height or diſtance of your Building, provided it be very great, ſo as to make the Spaces you meaſure large and fair. But it is convenient that the Plane on which you take the ſhade be not far from Perpendicular to the Sun, at leaſt not very Oblique, and that the Wall which caſts the ſhade, be ſtraight and ſmooth at top, and its Direction nearly Eaſt and Weſt, for Reaſons that will be well underſtood by a Reader ſkilful in the Doctrine of the Sphere. And it will be requiſite to take the Extream greateſt or leaſt deviation of the ſhadow of the Wall, becauſe the ſhade continues for a good time at a ſtand, without alteration, which will give the Obſerver leiſure to be aſſured of what he does, and not be ſurprized by the quick tranſient motion of the ſhade of a ſingle Point at ſuch a diſtance. The Principal Objection is, that the *Penumbra* or Partile ſhade of the Sun is in its extreams very difficult to diſtinguiſh from the true ſhade, which will render this Obſervation hard to determine nicely. But if the Sun be tranſmitted through a *Teleſcope*, after the manner uſed to take his *Species* in a *Solar Eclipſe*, and the upper half of the Object-glaſs be cut off by a Paper paſted thereon, and the exact upper Limb of the Sun be ſeen juſt Emerging out of, or rather continging the *Species* of the Wall, (the Poſition of the *Teleſcope* being regulated by a fine Hair extended in the *Focus* of the Eye-glaſs,) I am aſſured that the limit of the ſhade may be obtained to the utmoſt exactneſs: And of this I deſign to give a *Specimen* by an Obſervation to be made in *June* next, by the help of the High Wall of St. *Paul's* Church, *London*, of which ſome following Tranſaction may give an Account. In the mean time what I have premiſed may ſuffice to ſet others at work, where ſuch or higher Buildings are to be met with. I ſhall only Advertiſe, that the Winter-Tropick by this Method may be more certainly obtained than the *Summer's*, by reaſon that the ſame *Gnomon* does afford a much larger *Radius* for this manner of Obſervation.
